Roll No. .... Total Printed Pages - 8

F - 311

M.A./M.Sc. (First Semester) EXAMINATION, Dec. - Jan., 2021-22 MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] ***[Maximum Marks:80***

Note : Attempt all sections as directed.

Section - A

(Objective/Multiple Choice Questions)

(1 mark each)

1. Which is not a property of cantor set ?
   (A) Cantor ternary set is measurable and its measure is zero
   (B) Cantor set is equivalent to [0,1]
   (C) Cantor set is non empty
   (D) Cantor set is countable
2. Which of the following has different meaning?
   (A) Enumerable
   (B) Denumerable
   (C) Uncountable
   (D) Countable in finite

3. Which is countable set ?
   (A) Set of real number
   (B) Set of all irrational number
   (C) The unit interval [0,1]
   (D) Set of all integers.
4. Which of the following is not correct?
   (A) The set of all real numbers is not enumerable.
   (B) The set of all irrational numbers is not enumerable.
   (C) Every subset of countable set is countable.
   (D) The set of all real numbers in open interval (0,1) is enumerable.
5. Let $X=\{1,2,3,4,5\}$ then which of the following is topology on X?
   (A) $T_1=\{x,\phi,\{1,2,3\},\{2,3,5\}\{1,5\}\}$
   (B) $T_2=\{x,\phi,\{1\},\{2\},\{12\},\{4,5\}\{1,2,3,4,5\}\{1,4,5\}\}$
   (C) $T_3=\{x,\phi,\{1,2,3\}\{1,2,4\},\{1,3,5\}\}$
   (D) None of the above
6. If X is any set, T is collection of all subsets of X, then Topology (X,T) is -
   (A) A discrete topology.
   (B) Trivial topology.
   (C) Indiscrete topology.
   (D) None of the above.

7. Let X and Y are topalogical space. A function $f: X \rightarrow Y$ is continuous function.-

(A) If for each subset V of Y, the set $f^{-1}(v)$ is an closed subset of X

(B) If for each closed subset V of Y the set $f^{-1}(v)$ is an open subset of X.

(C) If for each open subset V of Y, the set $f^{-1}(v)$ is an open subset of X

(D) None of the above

8. Let (X,T) and (Y, U) be two topological spaces such that f: $X \rightarrow Y$ is homeomorphism then-

(A) f is not one - one and onto

(B) f is not onto

(C) f is not continuous

(D) f^{-1} is continuous

9. A topological space has acountable basis at each of its point is called -

(A) First countability axiom

(B) Second countability axiom

(C) Third countability axiom

(D) Fourth countability axiom

10. A subspace of normal space is-

(A) Need not normal

(B) Hausdorff

(C) Normal

(D) Need not Hausdorff

11. A subspace of regular space is -

(A) Hausdorff

(B) Disjoint

(C) Regular

(D) None of these.

12. Which of the following is not true?

(A) A regular T_1 space is called T_3 space

(B) A normal T_1 space is called $T_{3\frac{1}{2}}$ space

(C) Every discrete space is regular.

(D) Two sets A and B are separated if-

$A \neq \phi, B \neq \phi, \overline{A} \cap \overline{B} = \phi \;\; A \cap \overline{B} = \phi$

13. Every compact subset of Hausdorff space is -

(A) Closed set

(B) Open set

(C) Null set

(D) None of these

14. Every compact Hausdorff space is-

(A) Hausdorff

(B) Disjoint

(C) Normal

(D) None of these

15. Which of the following is not true -

(A) Continuous image of compact space is compact.

(B) A compact Hausdorff space is regular.

(C) A countably compact topological space has not a B.W.P.

(D) Every compact regular space is normal

16. Lebesgue covering lemma states that:

(A) Every open covering of a sequentially compact space has a lebesgue number.

(B) Every infinite bounded subset of R has at least one limit point.

(C) Every sequentially compact metric space is totally bounded.

(D) A closed subset of countably compact space is countably compact.

17. Which of the following is not ture ?

(A) A metric space is compact if fit is totally bounded and complete.

(B) A metric space X is compact iff metric space X is sequently compact.

(C) A compact metric space is separable

(D) Every compact subset of metric space is open and bounded.

18. Let E be a connected subset of a topological space X. If $E \subseteq A \subseteq \bar{E}$ then A is-

(A) Separable

(B) Connected

(C) Disconnected

(D) None of these.

19. A topological space X is locally connected iff-

(A) For every openset U of X, each component of U is closed in X.

(B) For every closed set U of X, each component of U is open in X.

(C) For every open set U of X, each component of U is open in X.

(D) None of above.

20. Which of the following is not true-

(A) The component of totally disconnected set in X are singleton sets in X

(B) A subspace of a real line is connected iff it is not an interval.

(C) The closure of connected set is connected.

(D) Two open subsets of a topological space are seperable iff they are disjoint.

Section - B

(Very Short Answer Type Question)

(1½ marks each)

Note : Attempt all questions.-

1. State schroeder - Bernstein theorem.
2. Define choice function.
3. Define co - countable topology.
4. Define base of topology.
5. Define second countable space.
6. Write an example of T_1 space.
7. Define open cover of a topological space.
8. State stone - cech compactification theorem.
9. Define locally connected space.
10. Define sequentially compact space.

Section - C

(Short Answer Type Questions)

(2 ½ marks each)

1. Prove that the power set of any set has cardinality greater than the cardinality of the set itself.
2. Define partially ordered relation with an example.
3. Prove that intersection of two topologies is also topology. But union of two topologies is not necessary a topology.
4. Let A be subset of topological space then $\overline{A} = A \cup D(A)$
5. Show that every metric space is first countable.
6. Prove that a topological space (X,T) is T_1 - space iff, every singleton set in X is closed.
7. Prove that continuous image of compact set is compact.

8. Prove that a compact space have Balzano weirstrass property.
9. Prove that every compact metric space is complete.
10. If A and B are conected sets which are separated, then AUB is connected.

Section - D

(Long Answer Type Questions)

(4 marks each)

Note: Attempt all questions.

1. State and prove Zorn's lemma.

   OR

   State and prove well ordering principle.
2. Show that a subspace of a topological space is itself a topological space.

   OR

   If (X T1) and (Y T2) be two topological space and $f: X \to Y$ is bijective mapping then following statements are equivalent.

   (a) *f* is a homeomarphism.

   (b) *f* is continuous as well as open.

   (c) *f* is continuous as well as closed.
3. State and prove urysohn's lemma.

   OR

   State and prove Tietze extension theorem.
4. A subset of R^n is compact iff it is closed and bounded.

   OR

   State and prove Lebesque covering lemma.
5. Prove that compact metric space is separable.

   OR

   Prove that subspace of a real line is connected iff it is an interval.