Roll No. ....

D–3752

M. A./M. Sc. (Previous) EXAMINATION, 2020

MATHEMATICS

Paper Second

(Real Analysis)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define Riemann-Stieltjes integral. If :

$$f_1 \in R(\alpha)$$

and $$f_2 \in R(\alpha)$$

on $[a, b]$, then show that :

$$f_1 + f_2 \in R(\alpha)$$

and $$\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha$$

(b) Define rectifible curve. If $Y : [a, b] \to R^k$ be a curve and $c \in (a, b)$, then show that :

$$\wedge_y(a, b) = \wedge_y(a, c) + \wedge_y(c, b)$$

(c) If $f \in R(\alpha)$ on the interval $[a, b]$, then show that :

$$m[\alpha(b) - \alpha(a)] \le \int_a^b f\, d\alpha \le M[\alpha(b) - \alpha(a)]$$

where m and M are bounds of the function f.

Unit—II

2. (a) State and prove Weierstrass's M-test. Show that the sequence $\{f_n\}_{n=1}^{\infty}$ where $f_n(x) = \dfrac{nx}{1 + n^2x^2}$ does not converge uniformly on R.

(b) State and prove Tauber's theorem on power series.

(c) State and prove the Weierstrass's approximation theorem.

Unit—III

3. (a) State and prove the implicit function theorem.

(b) Define Jacobian for two functions. If u_1, u_2 are functions of y_1, y_2 and y_1, y_2 are functions of x_1, x_2, then :

$$\frac{\partial(u_1, u_2)}{\partial(x_1, x_2)} = \frac{\partial(u_1, u_2)}{\partial(y_1, y_2)} \cdot \frac{\partial(y_1, y_2)}{\partial(x_1, x_2)}$$

(c) Find the maximum and minimum values of the function :

$$f(x, y) = 2x^2 - 3y^2 - 2x$$

subject to constraint $x^2 + y^2 \le 1$.

Unit—IV

4. (a) If A be any set and $E_1, E_2, \ldots, E_n$ a finite sequence of disjoint measurable sets, then show that :

$$m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$$

(b) State and prove the Jordan Decomposition theorem for a function of bounded variation.

(c) If f be a bounded and measurable function defined on $[a, b]$ and if :

$$F(x) = \int_a^x f(t)\, dt + F(a)$$

then show that $F'(x) = f(x)$ almost everywhere in $[a, b]$.

Unit—V

5. (a) If (X, B, μ) be a measure space, $E_i \in B$, $\mu(E_1) < \infty$ and $E_i \subset E_{i+1}$, then show that :

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu(E_n)$$

(b) State and prove Holder's inequality for L^p-spaces.

(c) Prove that L^p-space is a normed linear space.